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(1, 2) –VERTEX DOMINATION IN FUZZY LINE GRAPHS

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Abstract

A (1,2) - dominating set in a fuzzy graph G = (V,E) is a set S having the property that for every vertex v in V – S, there is at least one vertex in S at distance 1 from v and a second vertex in S at distance almost 2 from v. The minimum cardinality of (1,2) - dominating set in fuzzy graph G is called the (1,2) - domination number in G and we denote it by $\gamma(1,2)$.The fuzzy line graph L(G) of a fuzzy G =(V.E) is a graph with vertex set E(G) in which two vertices are adjacent if and only if the corresponding edges in G are adjacent. We introduce (1,2) - domination number of a fuzzy line graph and obtain some interesting results for the new parameter in fuzzy graph.

Keywords: Fuzzy graphs, (1,2)-dominating set in a fuzzy graph,(1,2)-domination number in a fuzzy graph, (1,2) - domination number in a fuzzy line graph.

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1.Introduction

In 1975, the notion of fuzzy graph and several fuzzy analogues of graph theoretical concepts such as paths cycles and connectedness are introduced by Rosenfeld [1]. Bhattacharya[2] has established some connectivity regarding fuzzy cut node and fuzzy bridges. The concept of domination in fuzzy graphs are introduced by A.Somasudaram and S.Somasundaram[6] in 1998. The concept of (1, 2) domination in graphs are introduced by N. Murugesan & Deepa.S.Nair in [4].In this paper, We analyze bounds on (1, 2) domination in fuzzy line graphs.

2. Preliminaries

Definition 2.1

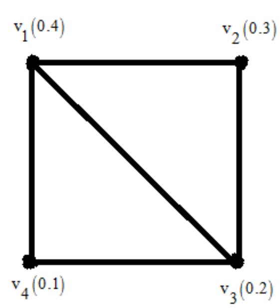
A fuzzy subset of a nonempty set V is mapping □ : V□ □ □ [0, 1] and A fuzzy relation on V is fuzzy subset of V x V. A fuzzy graph is a pair G: (□ ,□) where □ is a fuzzy subset of a set V and □ is a fuzzy relation on □ , where □ □ (u,v) ≤ □ (u) □ □ (v) □ □ u,v□ V

Definition 2.2

A fuzzy graph G=(σ, μ) is a strong fuzzy graph if μ(*u*,*v*) = σ (*u*) □σ (*v*) for all u, *v*∈V and is a complete fuzzy graph if μ(*u*,*v*) = σ (*u*) □σ (*v*) for all u, *v*∈V. The complement of a fuzzy graph G=(σ, μ) is a fuzzy graph $\bar{G}$ = ($\overline{\sigma}$,$\overline{\mu}$)where $\overline{\sigma}$=σ and $\overline{\mu}$ (*u*,*v*) = σ (*u*) □σ (*v*) - μ(*u*,*v*) for all u, *v*∈V

Definition 2.3

Let G=(σ, μ) be a fuzzy graph. Then D ⊆ V is said to be a fuzzy dominating set of G if for every *v*∈V-D, There exists u in D such that μ (*u*,*v*) = σ (*u*) □σ (*v*). The minimum scalar cardinality of D is called the fuzzy dominating number and is denoted by γ (G). Note that scalar cardinality of a fuzzy subset D of V is $|D| = \sum_{v \in V} \sigma(v)$



D={v_1},□ □ (G) =0.4

Definition 2.4

A dominating set D of a fuzzy graph G = (□ , □) is connected dominating set if the induced fuzzy sub graph <D> is connected. The minimum cardinality of a connected dominating set of G is called the connected domination number of G and is denoted by $□_c$ (G)

3.(1, 2) - Domination in fuzzy paths.

In this section we introduce (1, 2)- vertex domination in fuzzy line graphs. (1, 2)- vertex domination in fuzzy line graphs is denoted by (1, 2)- domination in fuzzy line graphs and corresponding some results.

Definition 3.1

A (1,2) - dominating set in a fuzzy graph G = (V,E) is a set S having the property that for every vertex v in V - S There is at least one vertex in S at distance 1 from v and a second vertex in S at distance almost 2 from v. The minimum cardinality of (1,2) - dominating set in fuzzy graph G is called the (1,2) - domination number in G and we denote it by γ(1,2).

Definition 3.2

The fuzzy line graph L(G) of a fuzzy G =(V.E) is a graph with vertex set E(G) in which two vertices are adjacent if and only if the corresponding edges in G are adjacent. we denote a cycle on n vertices by C_n, a path by P_n, a star fuzzy graph on n vertices by $K_{1,n}$.

Consider the following fuzzy paths,

P_3

{2} is a dominating set and {2,3} is a (1,2) - dominating set.

P_4

{2,3} is a dominating set and also {2,3} is a (1,2) - dominating set.

P_5

{2,4 } is a dominating set and {2,3,4 } is a (1,2) - dominating set

P_6

{2,5} is a dominating set and {2,3,4,5} is a (1,2) - dominating set.

P_7

{2,5,7} is a dominating set and {2,3,4,5,6} is a (1,2) - dominating set.

P_8

{2,5,7} is a dominating set and {2,3,4,5,6,7} is a (1,2) - dominating set.

P_9

{2,5,8} is a dominating set and {2,3,4,5,6,7,8} is a (1,2) - dominating set.

P_{10}

{2,5,8,9} is a dominating set and {2,3,4,5,6,7,8,9} is a (1,2) - dominating set.

From the above examples we have the following theorem.

Theorem 3.3(1,2) - dominating vertices of a path fuzzy graph P_n, for $n \geq 4$ is n-2.

Proof: Let P_n be a path with n v_1, v_2, ...,v_n. Then v_2, v_3, ...,v_{n-1} are adjacent to two vertices ,v_1 and v_2 are adjacent to one vertex. That is n-2 vertices are adjacent to two vertices. Each vertex v_1 is adjacent to v_{i+1}. Therefore vertices, v_i's are at distance one from v_{i+1}. Each vertex v_{i+2} is at distance 2 from v_i. So to form a (1,2) - dominating set we have to include all those vertices are adjacent to two vertices. But there are n-2 vertices are adjacent to two vertices. Hence (1,2) - dominating vertices is n-2.

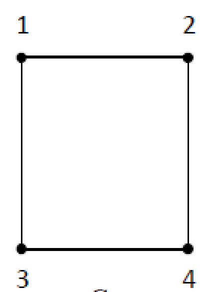
Theorem 3.4 The dominating vertices of the path P_n is less than (1,2) dominating vertices.

Proof: consider the above examples, we have dominating vertices of a path fuzzy graph P_n is $\left\lceil \frac{n}{3} \right\rceil$

In a fuzzy graph G, dominating vertices is less than or equals (1,2) dominating vertices. Let G be a fuzzy graph and D be its dominating set. Then every vertex in V-D is adjacent to a vertex in D. That is, in D, for every vertex u, there is a vertex which is at distance 1 from u. But it is not necessary that there is a second vertex at distance at most 2 from u. So if we find a (1,2)-dominating set, it will contain more vertices or at least equal number of vertices than the dominating set. So the dominating vertices is less than or equal to (1,2)- dominating vertices. In particular, for paths dominating vertices is less than (1,2) dominating vertices. Hence the theorem.

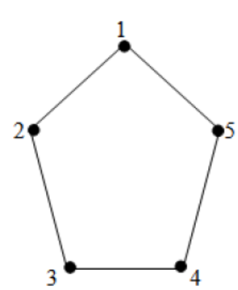
4. (1,2)-Domination in fuzzy cycles

Consider the following fuzzy cycles



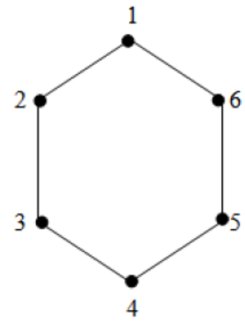
C_4

{2,3} is a dominating set and {2,3} is a (1,2) - dominating set.



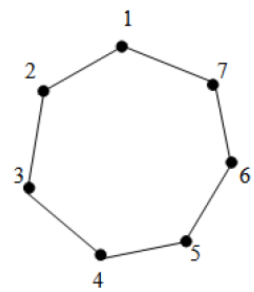
C_5

{1,3} is a dominating set and {1,3,4} is a (1,2) - dominating set.



C_6

{1,4} is a dominating set and {1,2,4} is a (1,2) - dominating set.



C_7

{1,4,6} is a dominating set and {1,3,4,7} is a (1,2) - dominating set.

From the above examples we have the following theorem.

Theorem 4.1 For cycle C_n, $n \geq 4$, (1,2) - dominating vertices is $\frac{n}{2}$ if n is even and (1,2) - dominating vertices is n-2 if n is odd.

Proof: Every cycle C_n have n vertices and n edges in which each vertex is adjacent to two vertices. That is each vertex dominates two vertices.

Case 1: n is even.

Let v_1, v_2, ...,v_n be the vertices of C_n. v_1 and v_n are adjacent. Also v_1 is adjacent to v_2, v_2 is adjacent to v_3 and so on. Each v_i, $1 < i < n$ is not adjacent to v_{i+2}, v_{i+3}, v_{i+4},..., v_{n-1}. Let us construct a (1,2) - dominating set. If we take the vertex v_1,v_1 is adjacent to v_2 and v_n and non-adjacent to all other n-2 vertices. v_3 and v_{n-1} are at distance 2 from v_1. So we have to take v_2 and any one of v_3, v_{n-1} in the set. If we take v_2, v_2 is adjacent to v_1 and v_3 and non- adjacent to v_4, v_5,

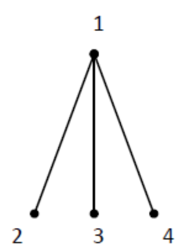
... v_n. That is n-3 vertices and v_4 and v_{n-2} are at distance 2 from v_2. Similarly we can proceed up to all the n vertices. Finally we get a (1,2)- dominating set containing v_1, v_2, v_4, v_6,.., v_{n-2}. Hence (1,2) - dominating vertices is $\frac{n}{2}$ if n is even

Case 2: n is odd

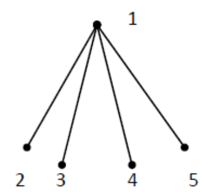
If n is odd, we remove one vertex v_1, then the other n-1 vertices form a path P_{n-1} and n-1 is even. But (1,2) - dominating vertices of P_{n-1} is n- 3. These n-3 vertices and the vertex v_1 from a (1,2) dominating set. Hence the cardinality of the (1,2) dominating set is n- 3+1. that is n-2. Hence (1,2) - dominating vertices is n-2 if n is odd

5. (1,2)-Domination in fuzzy star graphs

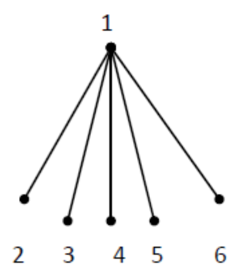
Consider the following fuzzy star graphs.



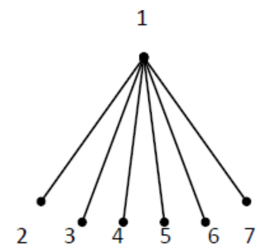
{1} is a dominating set and {1,2} is a (1,2) - dominating set.



{1} is a dominating set and {1,2} is a (1,2) - dominating set.



{1} is a dominating set and {1,2} is a (1,2) - dominating set.



{1} is a dominating set and {1,2} is a (1,2) - dominating set.

Theorem 5.1 For any fuzzy star $K_{1,n}$, (1,2) - dominating vertices is 2.

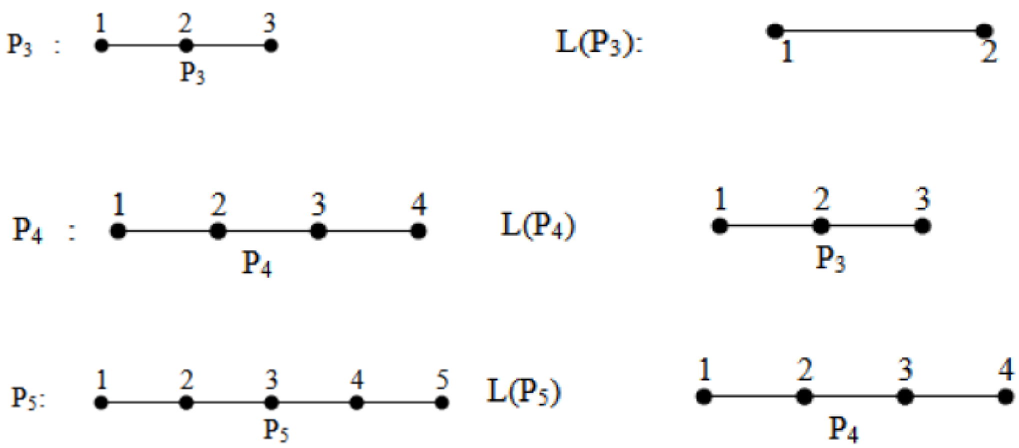
Proof:

In a fuzzy star $K_{1,n}$, there are n+1 vertices v, v_1, v_2, ...,v_n. v is adjacent to all other vertices v_1, v_2, ...,v_n. {v_1, v_2, ...,v_n} form an independent set. Each of v_1, v_2, ...,v_n are at a distance 1 from v and each of v_2 , v_3,...,v_n are at a distance 2 from v_1.So we can form a (1,2) dominating set as {v, v_1}. Hence (1,2) - dominating vertices is 2.

6.(1,2) - Domination in the fuzzy line graph of P_n, C_n, $K_{1,n}$.

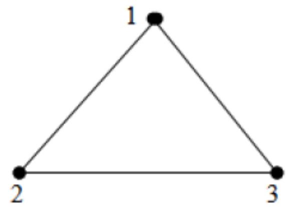
In this section first we discuss the fuzzy line graphs of paths, cycles and star graphs.

Consider the paths and the corresponding line graphs

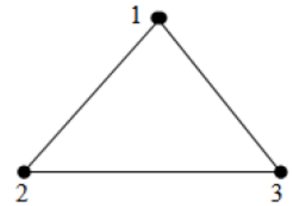


Next consider the cycles and the corresponding fuzzy line graphs

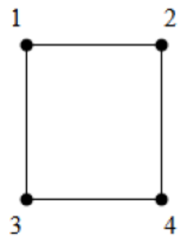
C_3:



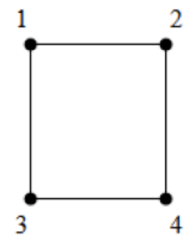
$L(C_3)$



C_4:

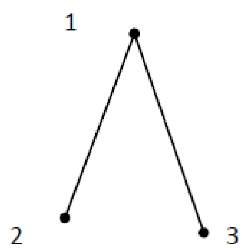


$L(C_4)$

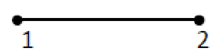


Consider the following fuzzy star graphs and the corresponding fuzzy line graphs

$K_{1,2}$:

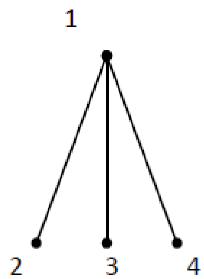


$L(K_{1,2})$

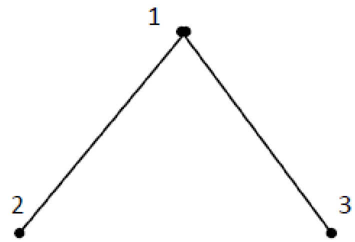


P_2

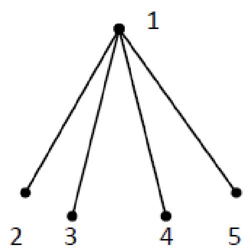
$K_{1,3}$:



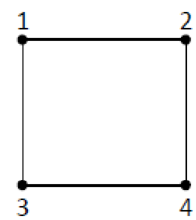
$L(K_{1,3})$



$K_{1,4}$:



$L(K_{1,4})$



Theorem 6.1

(1,2) dominating vertices of $L(P_n)$ is n-3.

Proof : P_n has n vertices and n-1 edges and L(Pn) is P_{n-1} with n-1 vertices and n-2 edges. Then by theorem 3.1, (1,2) - dominating vertices of P_{n-1} with n-3. Hence (1,2) - dominating vertices in the fuzzy line graph of $L(P_n)$ is n-3

Theorem 6.2 (1,2) - dominating vertices of $L(C_n)$ is $\frac{n}{2}$ if n is even and (1,2) - dominating vertices of $L(C_n)$

is n-2 if n is odd.

Proof: The fuzzy line graph of Cn, L(Cn) is Cn itself. So we can apply theorem 6.1 Hence (1,2) - dominating

vertices of $L(C_n)$ is $\frac{n}{2}$ if n is even and (1,2) – dominating vertices is n-2 if n is odd

Theorem 6.3 (1,2) - dominating vertices of L(K1,n) is same as that of C_n.

Proof: The fuzzy line graph of K1,n is Cn. Then by theorem 6.2, (1,2)- dominating vertices of L(K1,n) is $\frac{n}{2}$ if n is even and is n-2 if n is odd.

7. Conclusion

(1,2) –vertex domination in fuzzy line graph is defined. Theorems related to this concept are derived and the relation between (1,2) –vertex domination in fuzzy graph and (1,2) - vertex domination in fuzzy line graphs are established.

Reference

[1]. Rosenfeld A, Zedeh L A, Fu.K.Tanaka K S , Fuzzy sets and their Application to cognitive and Decision processes Academic press, Newyork (1975) ,77-95.

[2]. Bhattacharya, Some remarks on fuzzy graphs, Pattern Recognition letter 6,297-302.

[3]. Haynes T W, Hedetniemi S T and Slater P J, Fundamentals of domination in Graphs, Marcel Dekker,New York, 1998.

[4]. Murugesan N and Deepa S Nair, (1,2) - domination in Graphs,J. Math.Comput. Sci., Vol.2, 2012, No.4, 774- 783.

[5] .Murugesan N. and Deepa S. Nair, The Domination and Independence of Some Cubic Bipartite Graphs, Int. J.Contemp. Math Sciences, Vol.6, 2011, No.13,611-618.

[6].Somasundaram A and Somasundaram S,Domination in Fuzzy graph-I, Pattern Recognition letter 19(9) (1998) ,77- 95.

[7].Sarala N,Kavitha T,Triple connected domination number of fuzzy graph, International Journal of Applied Engineering Research, Vol. 10 No.51 (2015)914-917

[8]. Sarala N, Kavitha T,Connected Domination Number of Square Fuzzy Graph ,IOSR-JM,Volume10, Issue 6

Vel III (2014), 12-15

[9].Sarala N,Kavitha T,Neighborhood and efficient triple connected domination number of a fuzzy graphIntern. J. Fuzzy Mathematical Archive Vol. 9, No. 1, 2015, 73-80

[10].Sarala N, Kavitha T ,Strong (Weak) Triple Connected Domination Number of a Fuzzy Graph, International Journal of Computational Engineering Research, Volume, 05 Issue, 11 2015 ,18-22

[11] Sarala N,Kavitha T, (1, 2) domination in Fuzzy graphs, International Journal of Innovative Research in Science, Engineering and Technology, Vol. 5, Issue 9(September 2016),PP:16501-16505.

[12] Sarala N,Kavitha T, complete and complement domination in interval valued fuzzy graphs", International Journal of Science and Research, Volume 5 Issue 8(August 2016) PP:2046-2050.